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# BLOCK PERMANENT MAGNET SYSTEM MAGNETIC FIELD DETERMINATION

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## ABSTRACT

The paper presents the magnetic field calculation of permanent magnet system, which component parts are block magnets homogeneously magnetized in known direction. Method used in the paper is based on a system of equivalent magnetic dipoles. The results that are obtained using this analytical method are compared with results obtained using COMSOL Multiphysics software. Magnetic field and magnetic flux density distributions of permanent magnet are also shown in the paper.

**Index Terms** - Magnetic field, Permanent magnet, Magnetic dipole

## 1. INTRODUCTION

Permanent magnetism is one of the oldest continuously studied branches of the science. There are many properties of permanent magnets that are taken into consideration when designing a magnet for a certain device. Most often the demagnetization curve is the one that has the greatest impact on its usability. Curve shape contains information on how the magnet will behave under static and dynamic operating conditions, and in this sense the material characteristic will constrain what can be achieved in the device's design. The B versus H loop of any permanent magnet has some portions which are almost linear, and others that are highly non-linear.

Magnetic materials are vital components of most electromechanical machines. An understanding of magnetism and magnetic materials is therefore essential for the design of modern devices. The magnetic components are usually concealed in subassemblies and are not directly apparent to the end user. Permanent magnets have been used in electrical machinery for over one hundred years. Scientific breakthroughs in materials study and manufacturing methods from the early 1940's to the present have improved the properties of permanent magnets and

made the use of magnetic devices common. This work is motivated by the need for different shaped permanent magnets in great number of electromagnetic devices.

Determination of the magnetic field components in vicinity of permanent magnets, starts with presumption that magnetization,  $\mathbf{M}$ , of permanent magnet is known. The following methods can be used in practical calculation:

- a) Method based on determining distribution of microscopic ampere's current;
- b) Method based on Poisson and Laplace equations, determining magnetic scalar potential; and
- c) Method based on a system of equivalent magnetic dipoles [1].

The third method that is mentioned in the paper for magnetic field calculation is based on superposition of elementary results obtained for elementary magnetic dipoles.

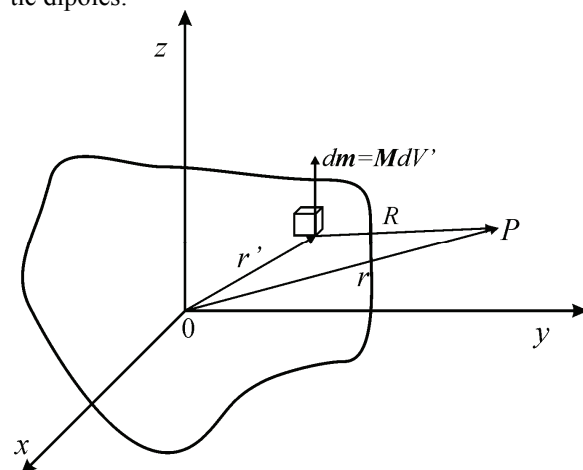


Figure 1. Elementary magnetic dipole.

Elementary magnetic dipole (Figure1) has magnetic moment

$$d\mathbf{m} = \mathbf{M} dV' . \quad (1)$$

This magnetic moment produces, at field point  $P$ , elementary magnetic scalar potential

$$d\varphi_m = \frac{1}{4\pi} \frac{\mathbf{R} d\mathbf{m}}{R^3} = \frac{1}{4\pi} \frac{\mathbf{R}\mathbf{M}}{R^3} dV', \quad (2)$$

where  $R = |\mathbf{r} - \mathbf{r}'|$  is distance from the point where the magnetic field is being calculated to elementary source, and  $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ .

After integration magnetic scalar potential is obtained as

$$\varphi_m = \frac{1}{4\pi} \int_V \frac{\mathbf{R} d\mathbf{m}}{R^3} = \frac{1}{4\pi} \int_V \frac{\mathbf{R}\mathbf{M}}{R^3} dV'. \quad (3)$$

Magnetic field vector can be expressed as

$$\mathbf{H} = -\text{grad } \varphi_m. \quad (4)$$

## 2. PROBLEM DEFINITION

The method that is described above is used for determining the magnetic field components of the block permanent magnet system presented in the Figure 2.

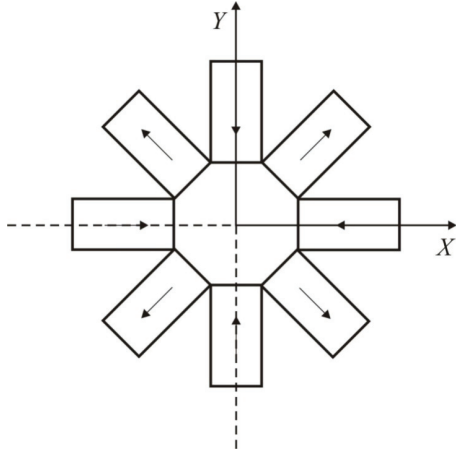


Figure 2. Block permanent magnet system.

Magnetic scalar potential of the whole system is the sum of magnetic scalar potentials obtained for each magnetized block.

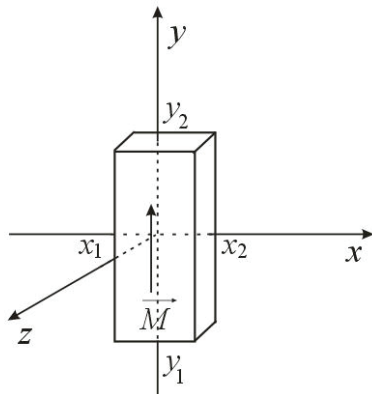


Figure 3. Block permanent magnet

To determine the magnetic field of the system the permanent magnet presented in the Figure 3 will be considered. It is homogeneously magnetized in longitudinal direction,

$$\mathbf{M} = M\hat{y}. \quad (5)$$

Outside the permanent magnet, magnetic scalar potential, at the field point  $P(x, y, z)$ , could be presented using the expression (3).

As magnetization has only  $y$  component, scalar product  $\mathbf{R} \cdot \mathbf{M}$  is formed as

$$\mathbf{R} \cdot \mathbf{M} = [(x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}]M\hat{y} \quad (6)$$

therefore,

$$\mathbf{R} \cdot \mathbf{M} = (y - y')M. \quad (7)$$

Distance from the point where the magnetic field is being calculated to elementary source is

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}. \quad (8)$$

Finally, substituting the expressions (7) and (8) in (3), magnetic scalar potential produced by a block magnet that is homogeneously magnetized in positive direction of  $y$  axis (Figure 3) is formed as

$$\varphi_m(x, y, z) = \frac{M}{4\pi} (V[y - y_2, x - x_1, x - x_2, z - z_1, z - z_2] - V[y - y_1, x - x_1, x - x_2, z - z_1, z - z_2]), \quad (9)$$

where function  $V$  has the following form:

$$\begin{aligned} V(a, x_1, x_2, z_1, z_2) = & x_2 \ln \frac{C_2}{C_3} + x_1 \ln \frac{C_1}{C_4} + z_1 \ln \frac{C_5}{C_8} + \\ & z_2 \ln \frac{C_6}{C_7} - 2|a| \text{Arctg} \frac{C_5 \cdot C_8 + a^2 + z_1^2 + z_1(C_5 + C_8)}{|a|(C_8 - C_5)} + \\ & 2|a| \text{Arctg} \frac{C_7 \cdot C_6 + a^2 + z_2^2 + z_2(C_6 + C_7)}{|a|(C_6 - C_7)} \end{aligned} \quad (10)$$

and

$$C_1 = z_1 + \sqrt{a^2 + x_1^2 + z_1^2}; \quad C_2 = z_2 + \sqrt{a^2 + x_2^2 + z_2^2};$$

$$C_3 = z_1 + \sqrt{a^2 + x_2^2 + z_1^2}; \quad C_4 = z_2 + \sqrt{a^2 + x_1^2 + z_2^2};$$

$$C_5 = x_1 + \sqrt{a^2 + x_1^2 + z_1^2}; \quad C_6 = x_2 + \sqrt{a^2 + x_2^2 + z_2^2};$$

$$C_7 = x_1 + \sqrt{a^2 + x_1^2 + z_2^2}; \quad C_8 = x_2 + \sqrt{a^2 + x_2^2 + z_1^2}.$$

With multiple translations and rotations of coordinate system, the block shown in the Figure 3 may be positioned like it's shown on Figure 2. to obtain suitable system.

If the original point is  $P(x, y, z)$  it can be translated to  $P'(x', y', z')$ . New coordinates are:

$$\begin{aligned}x' &= x - x_1, \\y' &= y - y_1, \\z' &= z.\end{aligned}\quad (11)$$

If the coordinate system is rotated clockwise around center point placed in the center of coordinate system new coordinates shall be

$$\begin{aligned}x' &= x \cos \alpha + y \sin \alpha, \\y' &= -x \sin \alpha + y \cos \alpha, \\z' &= z.\end{aligned}\quad (12)$$

In cases where system has  $N$  block permanent magnets, the magnetic scalar potential of the whole system is equal to sum of magnetic scalar potentials formed by all magnetized parts:

$$\varphi_m = \sum_{i=1}^N \varphi_{m_i}. \quad (13)$$

After determining magnetic scalar potential magnetic field component can be easily calculated using the expression (4).

### 3. NUMERICAL RESULTS

Distribution of magnetic field, in  $xOy$  plane, outside the permanent magnet is illustrated in the Figure 4. It is obtained using the analytical method for magnetic field determination.

Magnetic field distribution for the same system, obtained using the COMSOL Multiphysics software is presented in the Figure 5. Distribution of magnetic flux density is shown in the same figure with arrows and its intensity is presented with gradient of gray. Magnetization of each block in the system is 750kA/m.

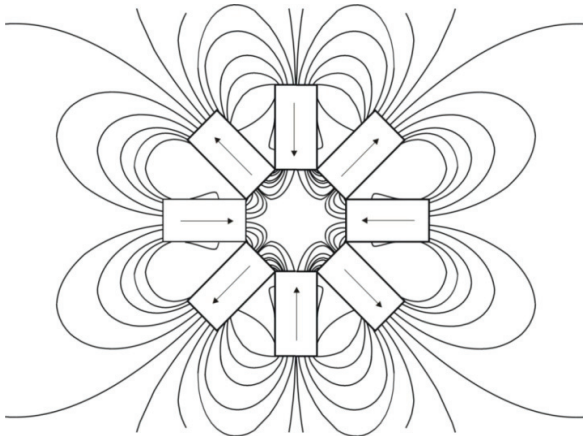


Figure 4. Distribution of magnetic field

Comparing these two figures it can be concluded that results of the analytical method are confirmed in

satisfactory manner using COMSOL Multiphysics software.

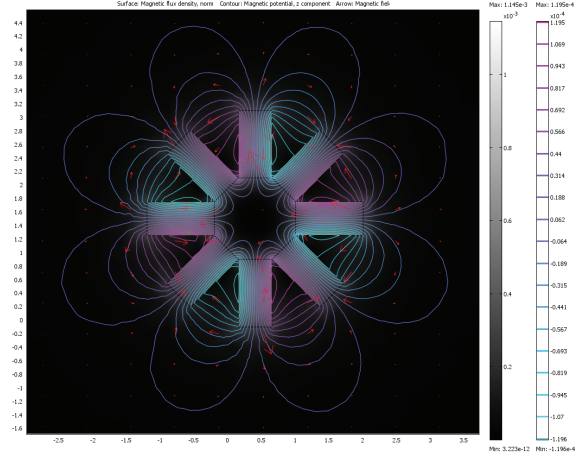


Figure 5. Distribution of magnetic field (COMSOL Multiphysics software)

### 4. CONCLUSION

The permanent magnet system which consists of block permanent magnets, homogeneously magnetized in known direction, is observed in the paper. Method that is used for magnetic field determination is based on superposition of results that are obtained for elementary magnetic dipoles. Magnetic field and magnetic flux density distribution of permanent magnet is also presented in the paper. Magnetic field lines have the same form and the same direction as magnetic flux density lines, outside the magnet. Results obtained by the analytical method are satisfactory confirmed using COMSOL Multiphysics software. This system may be applicable for approximation of the permanent magnets system found in rotation motors.

### 5. ACKNOWLEDGMENT

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